

Calculators are not allowed

Answer the following questions (all items are weighted equally at 5 points each):

1. Evaluate each of the following limits, if it exists:

(a) $\lim_{x \rightarrow 0} \frac{|\sin |x||}{x}$

(b) $\lim_{x \rightarrow 1} \frac{\sqrt{3x+1} - \sqrt{x+7}}{x-1}$

2. (a) Let

$$\frac{\sqrt{1+x^2}-1}{x^2} \leq f(x) - 2g(x) \leq \frac{2x - \sin x}{2x}, \text{ for } x \in (0, 1).$$

If $\lim_{x \rightarrow 0^+} g(x) = \frac{1}{4}$, then find $\lim_{x \rightarrow 0^+} f(x)$.

(b) Find the x -coordinate of all the points of discontinuities in $(-\infty, \infty)$ of

$$f(x) = \begin{cases} \frac{\sin(-x)}{x} & , \text{ if } x < 0 \\ \frac{1}{x} & , \text{ if } 0 < x < 1 \\ \frac{x^2 - x - 2}{x - 2} & , \text{ if } 1 \leq x \end{cases}$$

and classify the discontinuities of f as removable, jump, or infinite.

3. (a) Find the vertical and horizontal asymptotes, if any, of the function

$$f(x) = \frac{\sqrt{1-x}}{x|x-1|}$$

$$f(x) = \begin{cases} x^2 \sqrt{2 + \frac{1}{x}} + A & , \text{ if } x \neq 0 \\ 1 & , \text{ if } x = 0 \end{cases}$$

find the constant A such that f is continuous on \mathbb{R} . Determine whether $f'(0)$ exists or not.

4. (a) Use the definition of the derivative to find $f'(x)$, if $f(x) = \frac{1}{\sqrt{x}}$.

(b) Find $f'(x)$, if $f(x) = \frac{x^3 \tan x - 5x^2 + 4}{x \csc x - \sqrt{x} - 1}$.

(Good Luck)